Mathematics: The nature of the beast

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This paper has grown out of a research project concerned for participants' in and out-of-school mathematics. The project itself is a qualitative investigation into adult basic education students' experiential worlds. It has been designed to generate insights into supposedly innumerate adults' practices both in everyday contexts and in educational institutions. The presentation reports a cohort of educators', researchers' and theorists' responses to a particular out-of-school practice.

Mathematical learning and mathematical practices generate investigative efforts around the world. As a result, many concerned parties have pondered the nature of mathematics. Questions such as 'what is mathematics? What does mathematics look like? How can we say that a practice is mathematical? have been embedded within the heart of many investigations. For instance, George Joseph has asked 'what is mathematics? how is it created?...'(1994, p183): Hans Fruedenthal also asked the same question. Fruedenthal added 'don't look it up in the dictionary! When ever I did the answer was wrong' (Fruedenthal, 1991, p1). Mathematics - 'the science dealing with the measurement, properties, and relations of quantities...' (Bernard, 1984, p 645). Perhaps I should have heeded Fruedenthal.

Whether for theoretical or functional purposes (or combinations of the two), the quest for a clearer notion of the nature of mathematics continues. After all, it has been noted that, 'any account of teaching and learning...needs to consider the nature of the knowledge to be taught' (Driver, Asoko, Leach, Mortimer & Scott, 1994, p5). This is most assuredly true in the case for mathematics where many people (unfortunately many students) have images of mathematics that tend to debilitate the construction of their relational understandings. For instance, many students believe that mathematics enjoys an independent existence, is a set of hard and fast facts, and can be transmitted in perfect entirety from articulate expert to clever, receptive novice. Investigations into and explications of the nature of mathematical ways of knowing and acting stand to contribute to overcome such beliefs. This is essential; misinformed views can have far reaching effects. For example, Ernest has stated that 'in order to maintain [take up] a critical perspective we need 'a re evaluation of the nature of mathematics...'.otherwise we help to 'serve a social aim, the reproduction of social hierarchy and the associated distribution of privileges' (1994, p3).

A clearer idea of the nature of mathematics may also enhance educators' attempts to access students' prior knowledges. Students' prior ways of knowing and acting mathematically may have been constructed in out-of-school settings. Recognising the mathematising in students' prior/out-of-school practices is surely dependant on the ability to recognise the features and nature of informal mathematics. (This does not mean to say that other ways of knowing do not contribute to students' constructions of mathematical concepts. For example, one must never subordinate the role that language plays in the development of mathematical ideas).

Understanding the nature of mathematics should also underpin assessments of both mathematical concepts and mathematical practices. Indeed, Clarke has stated that 'the assessment practices we advocate should constitute the most explicit articulation of our values, our knowledge and *our beliefs about what should comprise mathematical activity...*' (my italics, 1994b, ch 3). Given this, it follows that those of us who assess students' mathematics should have at least begun to have established informed beliefs about the nature of mathematical knowledges. We simply cannot make judgements based upon students' abilities to replicate class teachers' mathematical knowledge. This merely confirms absolutist views of mathematics. In contrast, if we attempt to substantiate the assumption that mathematics is 'humanely constructed, corrigible, revisable, results from human inquiry, inseparable from other knowledge, is value laden, and an inseparable part of the whole human fabric of thought, language and life' (Ernest, 1994, p3) we indubitably need a framework through which to view idiosyncratic mathematics.

Mathematics has been called a verb (Schminke & Arnold, 1971), a language (eg Clarke, 1994a, workshop 4), and a tool kit (Greeno, 1991, Seely Brown, Collins & Duguid, 1987). Mathematics may well be all of these things. However, these labels do not help to answer the following question. What makes particular practices and understandings mathematical?

Birth of an inquiry

My research certainly requires a lens through which to view mathematical ways of knowing and acting. The investigation is concerned with a sample of adult basic education students' in and out-of-school mathematical practices. The students concerned have volunteered to return to educational institutions to upgrade/relearn formal mathematics. The majority of participants consider themselves to be innumerate. Research efforts have begun to generate a pool of out-of-school practices. One such practice gave rise to the discussions that form the body of this paper. The following example dialogue grew out of a student's recount: she repaired a broken, glass table top with a piece of ply found at a building site.

S1 I'll have to think of this now... okay, how I did it?

I: Yes, just exactly how you did it.

S1 **(laughs)** Okay, it might take a while. To know, to work out um, an approximate measurement of something I need to, if I don't have a, if I don't have a measuring tape handy with me...like in my pocket or something like that and I don't...and I don't...the only other alternative is to visualise. So in my mind, I visualise the um, an accurate, an accurate...no an estimation of the same size of the object. Okay?

I: Your coffee table?

S1: Which in this case is my coffee table **(laughs).**

I: When you stopped by that house how did you know that that piece of wood would be okay?

S1 I've told you.

I: Just by visualising?

S1: Visualising, yeah.

I: What were you thinking **(when you were wondering whether that piece of wood would fit your table?)**

S1: Oh whether it would be too big or too small, whether it would fit. Whether the wood was too thin, not thick enough and when you put something on it...would it sag, just hang there?

I: When you said this much over. What were you thinking then?

S1: Oh about this much over (**As the student talked she held out one hand and gestured with her thumb and finger to show that the table top was about two and one half inches larger in area than the table's frame.**) Approximating, not measuring

I: What inches? centimetres?

S1: No, no, no. Not measuring, visualising.

Gathered thoughts

As anticipated and welcomed, my appeal for colleagues' deliberations elicited a variety of responses. Although I have received hoards of valued opinions, I'm only able to present a few. It should be noted, from the outset, that the majority of replies arrived via email and are thus (1) written in informal genres and (2) off-the-cuff (although thoughtful) replies. Given this, italicised labels rather than correspondents' names well be used as referencing mechanisms.

A number of people denied that mathematising supported this out-of-school practice. For example, *A* stated that the student's practice was a 'purely sensorimotor affair and...need not have involved any conception of a number. Nor [did] it have to involve counting or measuring'. He then added that

> mathematics is a domain governed by rules that involve units and counted pluralities (arithmetic) as well as (in higher forms of math) logic and abstract spatial constructs. If you are not sticking to the rules, you are not doing mathematics.

I agree that the student's practice did not appear to involve number or counting. To my mind, these are not contentious issues. Points to ponder are whether the student was measuring? was she merely visualising? what renders one mathematical and the other not? Most obviously, in a comparison of a certain quantification with another, a participant's use of conventional/non conventional units would help to distinguish visualisation from measurement. Notwithstanding, can we state that visualisation is not mathematical? If so, does it follow that tessellating is not a mathematical activity? As stated, *A* also suggested that 'if you are not sticking to the rules, then you are not doing mathematics'. One wonders whether the student who applies incorrect rules for multiplying a two digit number by another two digit number is doing mathematics? If a rule can be defined as a way of acting/thinking that produces a desired, expected effect, then the student may well have been following a self-constructed, intuitive rule. In fact, she may always use visualisation to size concrete objects.

B stated that 'mathematics consists of abstraction: the finding of patterns and similarities in the midst of diversity'. I would argue that my student may have undertaken such a search: she found a piece of ply with a similar shape and size to her original coffee table top from among many other building materials. She also had to have undertaken a mental comparison of size: both of these practices can be allied with base objectives in Level 1A of the newly structured National TAFE curriculum. In fact, Outcome 2 in Level 1A states that students should be assessed to determine if they can 'identify an object which is specified by one specific attribute, eg shape, colour, size' (Queensland Department of Technical and Further Education, Training and Employment, 1994, p539). We must set aside the fact that this is a fairly rudimentary ability: is it, or is it not mathematical? The fact that the processes are cited in curriculum statements should not necessarily convince us.

B goes on to say that he sees 'most everyday cognition, ethnomathematics, situated cognition as possible starting points for mathematics, not necessarily mathematics'. I agree that many out-of-school ways of knowing and acting could be used as foundations for formal, conventional mathematics but insist on adding a proviso. The fact that they could be used as precursors to in-school mathematics does not ensure that they are either non mathematical or mathematical.

Other colleagues wrestled with associated issues. For instance, *C* maintained that definitions of mathematics need to be considered through lenses which acknowledge/critique current power relations in the field of mathematics. She questioned 'if indeed this student is said to be doing maths, how does this maintain and reproduce the power of mathematics?' While this is most certainly a primary concern, it cannot be adequately dealt with in the confines of this paper.

D noted distinctions between my student's practice and more precise means of measuring. The student's practice could be said to be 'very personal, not so transportable and not so reliable (except with continued practice) while 'more exact means are rather exact, (minute, insignificant errors notwithstanding) socially replicable and reliable' *E* also alluded to the notion of socially replicable ways of knowing. He stated that the student's practice did not 'appear to be part of a system of knowing 'that was 'adaptable, generalisable and could accommodate new situations'. Perhaps these are criteria that are more suited to in-school mathematical practices than out-of-school mathematical practices? Or, perhaps I should state that these processes are certainly desired outcomes for the in-school learning of mathematics. The student has used similar ways of knowing and acting to size in a different situation. I know, for instance, that she recently hemmed her daughter's skirt by visualising rather than by measuring.

The majority of responses indicated beliefs that the student's practice was mathematical. *F* was most adamant; 'of course this is mathematics. The student is using visual reasoning - an important component of mathematical reasoning'. *G* stated that although numbers were not involved 'the student was clearly involved in an 'area' estimation...an estimation of size'. I quite agree but are all estimations of size mathematical? *G* also suggested that, 'the student [was] aware that it [was] an approximation rather than a precise measurement, a mathematically metacognitive skill'. The student indicated this when she stated 'if I don't have a, if I don't have a measuring tape handy with me...like in my pocket or something like that and I don't...and I don't...the only other alternative is to visualise'. Each aspect of mathematics that *G* alludes to 'can be found in the Mathematics Student Outcome Statements for Western Australia' and I hasten to restate, in the Module Descriptors produced by the Queensland Department of Technical And Further Education, Training and Employment.

H noted both that the student's practice evoked a 'stimulus opportunity' and [was] certainly mathematical, touching upon the topics we label as Measurement (area) and Space (visualisation) and also feature[d] the pedagogical features of estimation and connections to 'real' world contexts' *I* also connected the practice with measurement, estimation and visualisation, as did *J* who added, 'the student knows what she was doing, why she doing it and was pretty systematic about it'. *K* focussed on the role that estimation played in the student's practice and explained that while we may teach standard ways of estimating, any student can develop their own. The mathematics may be naive or sophisticated or 'alternative' but if a student can visualise and also estimate the difference between a table top and a frame then they are doing mathematics.

Each and every response appeared to reflect it's contributor's theoretical stance. Most notably, *L* explained the situated nature of her definitions and concepts of the word mathematics.

If I were talking to pre-service students I would position myself in the broadly accepted social discourse. Here this activity would be mathematical. On the other hand, 'in another context, with mathematics educators and researchers...I would argue that what the student was doing was not necessary mathematical...with mathematicians I would argue that there was not mathematics going on because there was no creative construction of, or manipulation of, ideas that would lead to that person's control over the environment.

M (who works within an ethnomathematical framework) stated that, from her perspective, the situation was mathematical. She quoted Pompeu who suggested that

ethnomathematics refers to any form of cultural knowledge or social activity characteristic of a social and/or cultural group that can be recognized by other groups such as 'Western' anthropologists, but not necessarily by the group of origin, as mathematical knowledge or mathematical activity

Many contributors to this paper have considered the student's activity to be mathematical. If we were to agree with Pompeu's definition we could state that her activity was ethnomathematical.

At first, I simply assumed that the activity was mathematical. It appeared to involve estimation, informal 'measurement', reasoning about size and shape as well as other processes. Colleagues' reactions prompted me to re examine my assumptions.

For example, *N* stated that

the student appeared to have been employing "several but not all features of 'measurement. Some missing criteria are:

- the dimensional properties are expressed in appropriate units of measurement
- which are obtained with a mediating device or process expressed in those units
- with comparison made in terms of the units of measurement rather than the objects from which the measurements were taken

N would prefer to call the activity pre-mathematical. Elsewhere, I have also called the activity pre-mathematical (Grier, in press). Let us consider alternatives. If the student had a tape measure and (1) previously measured the original frame and (2) measured the piece of ply and (3) determined to fit one to the other, no one would doubt that her activity was mathematical. Instead of examining the student's actual practice, it may prove to be valuable to examine this hypothetical option. Why would the use of a measuring tape increase the likelihood of mathematising? Is the use of a measuring tape necessarily more creative, more 'mathematical' than visualisation?

Although I realise that further research efforts may help to either reinforce or critique my own developing view, I conclude with my own opinion. In the sense that it's constitution did not fully compare to more formal measurement tasks, the student's activity was pre-mathematical. However, it may well be that some out-of-school mathematics are 'competencies which include some, though not all of the elements of mathematical operations' (*N*, 1995). If this is the case, then the student's practice was certainly an instance of out-of-school mathematics.

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The reference list

Bernard, J., (1989). The Pocket Macquarie Dictionary. Australia. Jackaranda.

Clarke, D., (1994a). Assessment Modelling Sophisticated School Mathematics: A Workshop. *New Directions in Assessment in Mathematics. Proceedings of the Fourth Annual Conference on Mathematics Teaching and Learning*. Centre for Mathematics and Science Education. Queensland University of Technology.

Clarke, D. (1994b). Assessment Alternatives in Mathematics. *New Directions in Assessment in Mathematics. Proceedings of the Fourth Annual Conference on Mathematics Teaching and Learning*. Centre for Mathematics and Science Education. Queensland University of Technology.

Driver, R., Asoko, H., Leach, J., Mortimer, E., Scott, P. (1994). Constructing Scientific Knowledge in the Classroom. *Educational Researcher*. Vol 23. No 7. pp 5-12

Ernest, P. (1994). A Critical Mathematics Education is Critical of both Mathematics and Education. Social *Issues In Mathematics Education*. Newsletter 1. pp 2-4.

Fruedenthal, H. (1990). *Revisiting Mathematics Education. China Lectures*. Dordrecht. Kluwer Academic Publishers.

Greeno, J. (1991).Number Sense as Situated Knowing in a Conceptual Domain. *Journal for Research in Mathematics Education.* Vol 21. No 3. pp 170-211.

Grier, S. (in press). What is mathematics? What is not mathematics? What may be mathematics. in Hunting, R., FitzSimons, G., Clarkson, P., Bishop, A. *Regional Collaborations in Mathematics Education.* 1995. RICMI Proceedings.

Joseph, G. (1991). *The Crest of the Peacock: Non-European Roots of Mathematics*. London. I.B. Tauris & Co. Ltd.

Schminke, C., Arnold, W. (1971). *Mathematics is a Verb: Options for Teaching*. New York. Dryden Press.

Seely Brown. J., Collins, A., Duguid, P. (1989). Situated Cognition and the Culture of Learning. *Educational Researcher*. January-February. pp 32-41.

Queensland Department of Education, Training and Employment. (1994). Module 101: Numeracy - Level 1A. Number Skills 1. *TAFE:TEQ Module Descriptors*. Queensland Department of Education, Training and Employment. pp 532-543